Chapter 4
Basic Building Blocks of the Cost of Equity Capital – Size Premium

Size as a Predictor of Equity Returns

The size effect is based on the empirical observation that companies of smaller size are associated with greater risk and, therefore, have greater cost of capital. The "size" of a company is one of the most important risk elements to consider when developing cost of equity capital estimates for use in valuing a business simply because size has been shown to be a *predictor* of equity returns. In other words, there is a significant (negative) relationship between size and historical equity returns – as size *decreases*, returns tend to *increase*, and vice versa.[4.1]

Traditionally, researchers have used market value of equity (market capitalization, or simply "market cap") as a measure of size in conducting historical rate of return studies. However, as we discuss later in this chapter, market cap is not the only measure of size that can be used to predict return, nor is it necessarily the best measure of size to use.

Much of the research of the size effect relies on the data provided by the Center for Research in Security Prices (CRSP) databases at the University of Chicago. The CRSP database includes U.S. equity total returns (capital appreciation plus dividends) going back to 1926.

The CRSP databases enabled researchers to look at stocks with different characteristics and analyze how their returns differed. One of the first characteristics that researchers analyzed was large-market-capitalization (large-cap) companies versus small-market-capitalization (small-cap) companies.

For example, a 1981 study by Rolf Banz examined the returns of New York Stock Exchange (NYSE) small-cap companies compared to the returns of NYSE large-cap companies over the period 1926–1975.[4.2] What Banz found was that the returns of small-cap companies were *greater* than the returns for large-cap companies. Banz's 1981 study is often cited as the first comprehensive study of the size effect.

4.1 This chapter is excerpted in part from Shannon P. Pratt and Roger J. Grabowski, *Cost of Capital: Applications and Examples* 5th ed. (Hoboken, NJ: John Wiley & Sons, 2014).

4.2 Rolf W. Banz, "The Relationship between Return and Market Value of Common Stocks", *Journal of Financial Economics* (March 1981): 3–18. This paper is often cited as the first comprehensive study of the size effect.

Possible Explanations for the Greater Returns of Smaller Companies

Some valuation analysts treat small firms as equivalent to scaled-down large firms. This is likely an erroneous assumption.

There are theoretical reasons for the greater returns of smaller companies (i.e., the "size effect"), which might include: (i) small stocks are less liquid (with higher associated transaction costs), (ii) small stocks are riskier and harder to diversify, (iii) small stocks have higher betas which often are underestimated, (iv) investors must do more analysis per dollar invested, (v) investment data is less available.[4.3]

Valuation analysts also cite more practical reasons that small firms have risk characteristics that differ from those of large firms. For example, large firms may have greater ability to enter the market of the small firm and take market share away. Large companies likely have more resources to "weather the storm" in economic downturns. Large firms can generally spend more cash on R&D, advertising, and typically even have greater ability to hire the "best and brightest". Larger firms may have greater access to capital, broader management depth, and less dependency on just a few customers. A larger number of analysts typically follow large firms relative to small firms, so there is probably more information available about large firms. Small firms have fewer resources to fend off competition and redirect themselves after changes in the market occur.[4.4]

Any one of these differences (not an all-encompassing list) would tend to *increase* investors' required rate of return to induce them to invest in small companies rather than investing in large companies.

The size effect is not without controversy, nor is this controversy something new. Traditionally, small companies are believed to have greater required rates of return than large companies because small companies are inherently riskier. It is not clear, however, whether this is due to size itself, or to other factors closely related to or correlated with size (e.g., liquidity).[4.5] The qualification that Banz noted in his 1981 article remains pertinent today:

> *"It is not known whether size* [as measured by market capitalization] *per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size."*

In this chapter, we first present empirical evidence for the size effect, followed by a discussion of common criticisms of the size effect.

4.3 Credit: Roger Ibbotson.

4.4 M. S. Long and J. Zhang, "Growth Options, Unwritten Call Discounts and Valuing Small Firms", EFA 2004 Maastricht Meetings Paper no. 4057, March 2004. Available at http://www.ssrn.com/abstract=556203.

4.5 Even after controlling for size, research suggests that liquidity is still a systematic factor and a predictor of returns. See Roger G. Ibbotson, Zhiwu Chen, Daniel Y.-J. Kim, and Wendy Y. Hu, "Liquidity as an Investment Style", *Financial Analysts Journal* Vol 69(3): 30–44, May/June 2013, and Roger G. Ibbotson, Ph.D. and Daniel Y.-J. Kim, Ph.D., "Liquidity as an Investment Style: 2018 Update". Copies available at www.zebracapm.com. Most recently (2019), Ibbotson and colleagues Thomas M. Idzorek, CFA, Paul D. Kaplan, CFA, and James X. Xiong, CFA published a new Chartered Financial Analyst® (CFA) Institute Research Foundation monograph entitled, Popularity: A Bridge Between Classical and Behavioral Finance (available for download at https://www.cfainstitute.org/en/research/foundation/2018/popularity-bridge-between-classical-and-behavioral-finance or go to the CFA website at cfainstitute.org and search for "popularity".

The Size Effect: Empirical Evidence

Summary statistics over the 1926–2018 period for CRSP NYSE/NYSE MKT/NASDAQ[4.6] deciles 1–10 are shown in Exhibit 4.1. As size (in this case, as measured by market cap) *decreases*, return tends to *increase*. For example, the annual arithmetic mean return of decile 1 (the largest-cap companies) was 11.04% over the 1926–2018 period, while the annual arithmetic mean return of decile 10 (the smallest-cap companies) was 19.80%. Note that this increased return comes at a price: risk (as measured by standard deviation) increases from 18.81% for decile 1 to 42.11% for decile 10. The relationship between risk and return is a fundamental principle of finance and for estimating the cost of capital.

Exhibit 4.1: Summary Statistics of Annual Returns (CRSP NYSE/NYSE MKT/NASDAQ Deciles) 1926–2018

Decile	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
1-Largest	9.31%	11.04%	18.81%
2	10.42%	12.66%	21.36%
3	10.91%	13.41%	23.23%
4	10.69%	13.60%	25.39%
5	11.22%	14.31%	26.04%
6	11.18%	14.59%	27.00%
7	11.42%	15.19%	28.86%
8	11.28%	15.77%	32.69%
9	11.34%	16.65%	36.84%
10-Smallest	12.95%	19.80%	42.11%

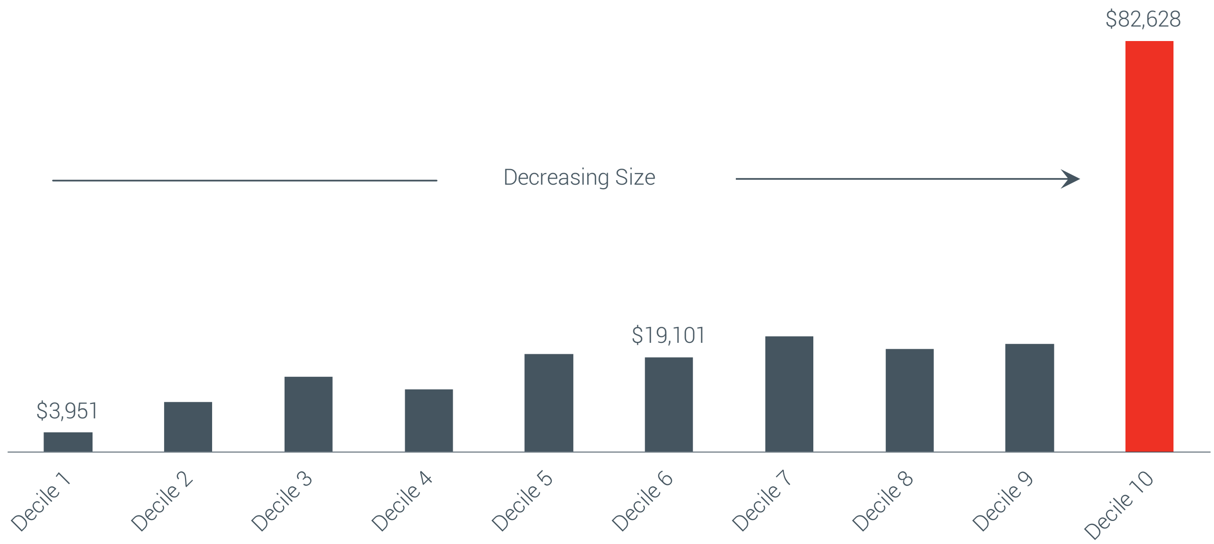
Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. CRSP NYSE/NYSE MKT/NASDAQ deciles 1–10. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

The Size Effect Over Longer Periods

Exhibit 4.2 illustrates the size effect. As size (measured by market cap in this case) *decreases*, return tends to *increase*. For example, an investment of $1 in CRSP decile 1 (comprised of the largest companies) at the end of 1925 would have grown to $3,951 by the end of 2018, and an investment in CRSP decile 6 (comprised of medium-sized companies) would have grown to $19,101. However, an investment of $1 in CRSP decile 10 (comprised of the smallest companies) would have grown to $82,628 over the same period.

[4.6] On October 1, 2008, NYSE Euronext acquired the American Stock Exchange (AMEX). The "NYSE MKT" is the former American Stock Exchange, or AMEX. The CRSP standard market-cap-based NYSE/AMEX/NASDAQ indices are now called the NYSE/NYSE MKT/NASDAQ indices.

Exhibit 4.2: Terminal Index Values of CRSP NYSE/NYSE MKT/NASDAQ Deciles 1–10
Index (Year-end 1925 = $1.00)
January 1926–December 2018



Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. CRSP NYSE/NYSE MKT/ NASDAQ deciles 1–10. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

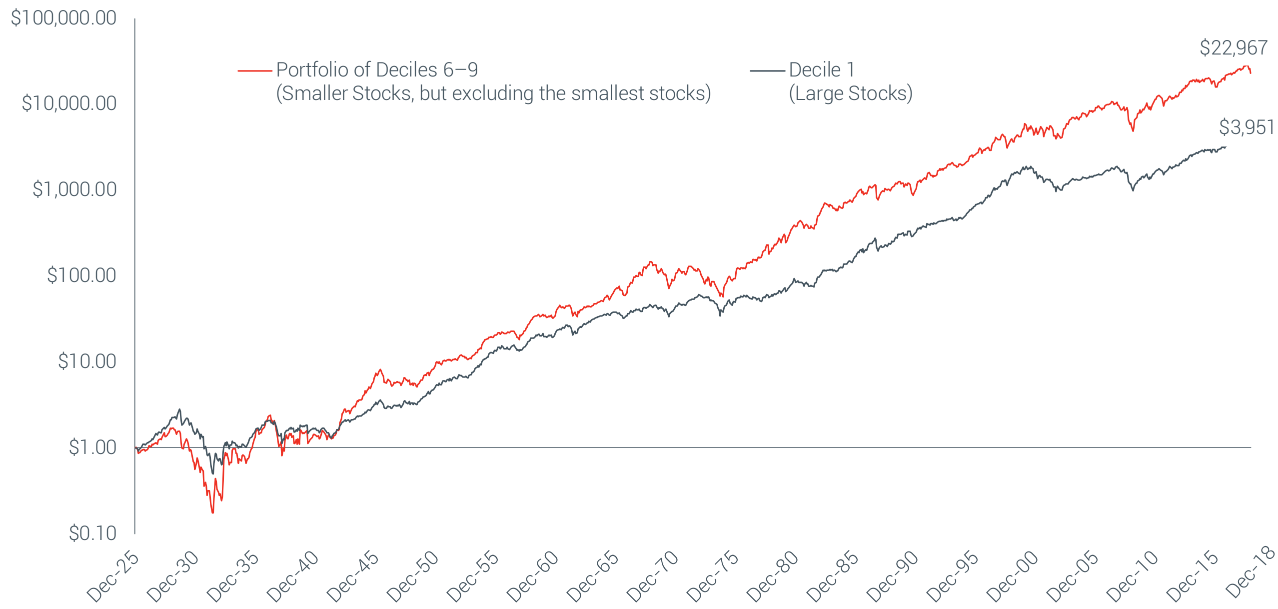
Exhibit 4.2 illustrates two other important concepts. The first is that the size effect is not "linear" – the size effect is clearly concentrated in the smallest-cap companies.[4.7]

The second is that over longer periods of time the size effect is *not* just evident for the smallest companies, but is evident for all but the largest groups of companies, including companies with a market capitalization in excess of several billions of dollars.

To illustrate this, decile 1 (large-cap companies) is compared to a portfolio comprised of equal parts of deciles 6–9 in Exhibit 4.3. An investment of $1 in decile 1 at the end of 1925 would have grown to $3,951 by the end of 2018, while an investment of $1 in a portfolio comprised of equal parts of deciles 6–9 at the end of 1925 would have grown to $22,967 by the end of 2018 (remember decile 10, which is comprised of the smallest-cap companies, is *excluded* from this analysis). Even with decile 10 *excluded*, the portfolio made up of deciles 6–9 outperformed large-cap companies over the 1926–2018 period.

[4.7] Some researchers have suggested that the size effect is concentrated in even smaller firms than discussed here. Horowitz, Loughran, and Savin found that if "...firms less than $5 million in value are excluded from the sample universe...", the size effect becomes insignificant, at least as measured over the 1963–1997 time period. Joel L. Horowitz, Tim Loughran, and N.E. Savin, "The disappearing size effect", *Research in Economics* (2000), 83–100.

Exhibit 4.3: Terminal Index Values of CRSP NYSE/NYSE MKT/NASDAQ Decile 1 and a Portfolio Comprised of equal parts of Deciles 6–9
Index (Year-end 1925 = $1.00)
January 1926–December 2018



Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. CRSP NYSE/NYSE MKT/NASDAQ deciles 1 and decile 6-9. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

Small-cap companies do not always outperform large-cap companies. As a matter of fact, small-cap companies' shorter-term behavior relative to large-cap companies can be quite erratic, so analyzing small-cap companies' performance relative to large-cap companies' performance over varying holding periods may be instructive in revealing longer-term trends.

In Exhibit 4.4, the percentage of periods in which small-cap companies outperformed large-cap companies is analyzed over 1-, 5-, 10-, 20- and 30-year holding periods. As the holding period is increased, small-cap companies tend to outperform large-cap companies in a greater number of periods. In other words, the *longer* small-cap companies are given to "race" against large-cap companies, the greater the chance that small-cap companies outpace their larger counterparts. For example, small-cap companies outperformed large-cap companies 81.9% of the time over all 20-year holding periods from January 1926 through December 2018. In contrast, large-cap companies outperformed small-cap companies only 18.1% over the same holding and time period.

Exhibit 4.4: Percentage of Periods that Small-cap Companies Outperform Large-cap Companies over 1-, 5-, 10-, 20-, and 30-year Holding Periods (1926–2018)

Holding Period	1-year	5-years	10-years	20-years	30-years
Small-cap Companies Outperform (%)	52.9%	55.9%	69.9%	81.9%	90.9%
Large-cap Companies Outperform (%)	47.1%	44.1%	30.1%	18.1%	9.1%

Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. Small-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ decile 10; Large-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ decile 1. The number of 1-, 5-, 10-, 20-, and 30-year holding periods over the January 1926–December 2018 time horizon is 1,105, 1,057, 997, 877, and 757, respectively. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

The Size Effect Tends to Stabilize Over Time

It may be instructive to examine the tendencies of small-cap stocks' performance versus large-cap stocks' performance over time periods with *fixed* starting dates and *variable* ending dates. This will help to see what happens as more time periods are added (and thus the importance of "unusual" time periods is diminished).

In Exhibit 4.5, the average difference in annual returns for small-cap companies minus large-cap companies was calculated for periods with fixed starting dates of 1926 (the first year data is available from CRSP), 1963 (the Risk Premium Report Study are calculated over the time period 1963–2018), and 1982 (the year following publication of Banz's 1981 article).[4.8]

On the far left side of Exhibit 4.5 for the series "Fixed Beginning Date Starting 1926", the first data point is the average difference in annual return for small-cap companies minus large-cap companies in 1926, the second data point (moving to the right) is the average difference in annual return for small-cap companies minus large-cap companies over the period 1926–1927, and then 1926–1928, etc., until the final data point on the far right is the average difference in annual return for small-cap companies minus large-cap companies over the period 1926–2018.

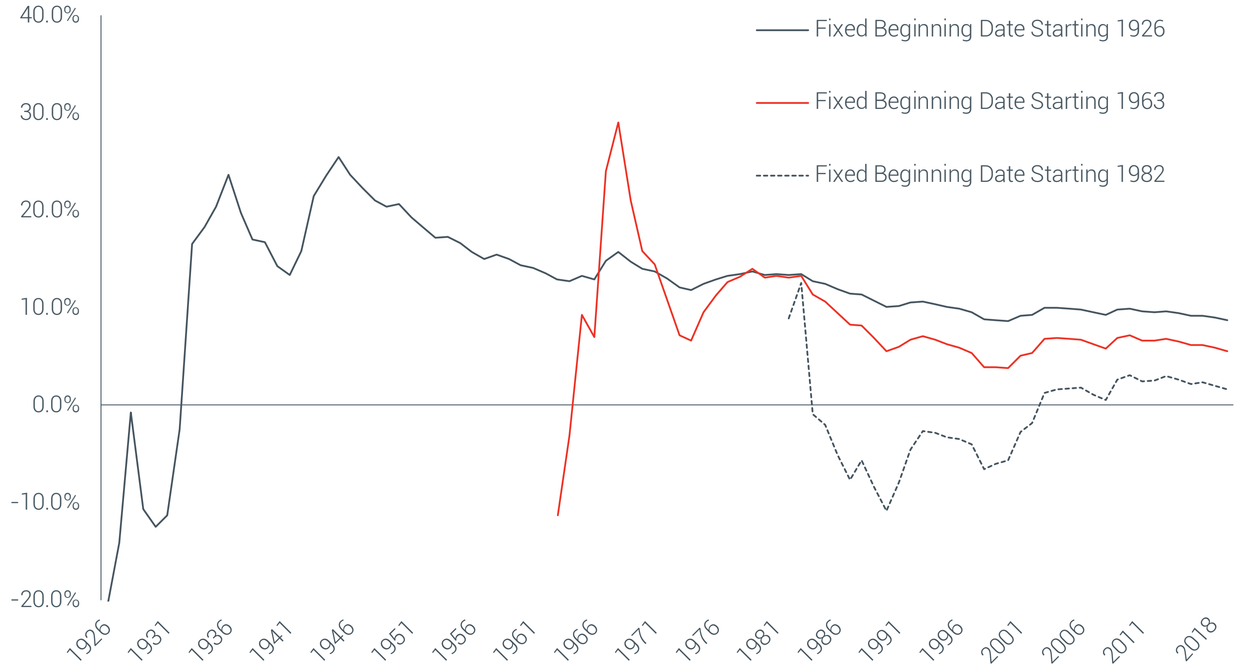
The same analysis is displayed for "Fixed Beginning Date Starting 1963", with the leftmost data point being the average difference in annual return for small-cap companies minus large-cap companies in 1963, and then (again, moving to the right) the average difference in annual return for small-cap companies minus large-cap companies over the periods 1963–1964, 1963–1965, etc., until the final data point on the far right is the average difference in annual return for small-cap companies minus large-cap companies over the period 1963–2018.

And finally, the same analysis for "Fixed Beginning Date 1982" is shown, with the leftmost data point being the average difference in annual return for small-cap companies minus large-cap companies in 1982, and the rightmost data point being the average difference in annual return for small-cap companies minus large-cap companies over the period 1982–2018.

4.8 Banz, Rolf W. "The Relationship between Return and Market Value of Common Stocks". *Journal of Financial Economics* (March 1981): 3–18. Banz's 1981 article demonstrated that smaller-cap stocks exhibited significantly greater performance over larger-cap stocks over the period from 1926 to 1975.

Exhibit 4.5 suggests that while the size effect measured over shorter time periods may be quite erratic (and even negative at times), there seems to be an overall tendency toward stability as time periods are added and the longer the period over which it is measured (regardless of the start date). Further, this stability seems to be reached in "positive territory" (the rightmost points in Exhibit 4.5), suggesting a positive size effect over time.

Exhibit 4.5: CRSP Decile 10 minus Decile 1, Average Difference in Annual Returns
Fixed beginning date, variable ending dates
1926–2018, 1963–2018, 1982–2018



Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. Small-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ decile 10; Large-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ decile 1. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

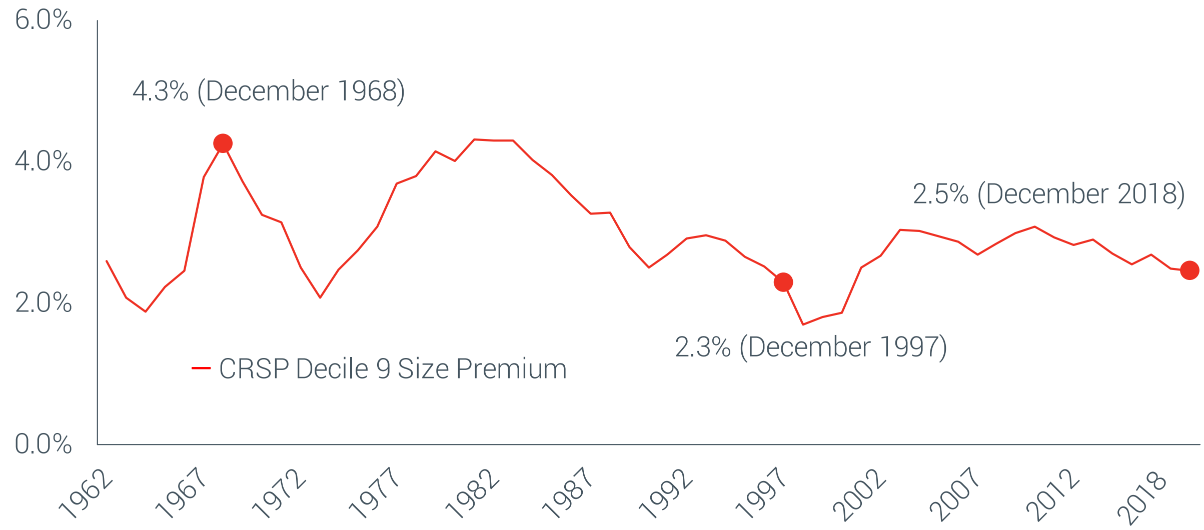
The Size Effect Changes Over Time

The variability of the size effect is illustrated in Exhibit 4.6. In Exhibit 4.6, the size premium for CRSP decile 9 (comprised of the smallest companies) is calculated as of each year-end from 1962–2018 using the same methodology and data set as is currently used in the Cost of Capital Navigator in the CRSP Deciles Size Study (and the same methodology and data set used previously in (i) the former *SBBI® Valuation Yearbook*, and (ii) Duff & Phelps' *Valuation Handbook – U.S. Guide to Cost of Capital*, and now in the online Cost of Capital Navigator, which replaced the *Valuation Handbook – U.S. Guide to Cost of Capital* in 2018).

For example, a hypothetical *Valuation Handbook* published in 1969 would have used data available from 1926–1968 to calculate CRSP decile 9's size premium, and this would have resulted in a size premium of approximately 4.3%. In a hypothetical *1998 Valuation Handbook – U.S. Guide to Cost of*

Capital, using data from 1926–1997, the size premium for CRSP decile 9 would have been approximately 2.3%. And, in the 2018 Cost of Capital Navigator using data from 1926–2018, the size premium for CRSP decile 9 is 2.5%.

Exhibit 4.6: CRSP Decile 9 Size Premium
Year-end 1962 to Year-end 2018



Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business; To learn more about CRSP, visit www.crsp.com. Small-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ decile 9. (ii) Morningstar, Inc. Used with permission. All rights reserved. The betas used as an input in calculating size premia were calculated using excess total returns over 30-day U.S. Treasury Bills. The market benchmark used in beta calculations is the S&P 500 total return index. Used with permission. All rights reserved. All calculations performed by Duff & Phelps, LLC.

These examples provide evidence that the size effect is *cyclical*. That cyclicality is part of the risk of small companies; if small size companies *always* performed better than large companies, small size companies would be *less* risky than large-cap companies, not riskier. This is true even though the expected returns are higher for small-cap companies in the long-term. By analogy, bond returns occasionally outperform stock returns. For example, over the 10-year period ending December 2011, long-term U.S. government bonds returned 133.2% and the S&P 500 Index return 33.4%, yet few would contend that over time the expected return on bonds is greater than the expected return on stocks.[4.9]

Criticisms of the Size Effect

The size effect is *not* without controversy, though, and various commentators question its validity. In fact, some commentators contend that the historical data are so flawed that valuation analysts can dismiss all research results that support the size effect. For example, is the size effect merely the result of not measuring beta correctly? Are there market anomalies that simply cause the size effect to appear? Is size just a proxy for one or more factors correlated with size, suggesting that valuation analysts should use those factors directly rather than size to measure risk? Is the size effect hidden because of unexpected events?

4.9 Source of underlying data: Morningstar *Direct* database. Calculations performed by Duff & Phelps, LLC.

Is the Size Effect the Result of Incorrectly Measuring Betas?

Some commentators have held that the size effect is in part a function of underestimating betas for troubled firms (which tend to populate the smaller deciles where size is measured by market cap). Including troubled companies could cause the size premium to be overestimated in the CRSP 10th decile and the subdeciles 10a (and its upper and lower halves 10w and 10x) and 10b (and its upper and lower halves 10y and 10z), which are populated with the smallest companies as measured by market cap.

The most commonly used size premia is derived based on an ordinary least squares regression (OLS) beta. We examine two alternative methods of calculating the beta in order to compute a size premia, sum betas and annual betas.

Effects of the Size Premia when Using OLS Betas, Annual Betas, and Sum Betas

Smaller companies generally trade more infrequently and exhibit more of a lagged price reaction (relative to the market) than do larger companies. One of the ways of capturing this lag movement is called “sum” beta. Sum betas are designed to compensate for the more infrequent trading of smaller company stocks.

The sum beta estimates are greater for smaller companies than OLS betas, which are derived using non-lagged market benchmark data. The net result of the *greater* sum betas (or greater annual betas) is *smaller* size premia.

In Exhibit 4.7a, OLS betas and sum betas are calculated for the CRSP standard deciles 1–10. The OLS betas and sum betas for the portfolios comprised of larger companies are approximately the same.

In Exhibit 4.7a, OLS betas, and sum betas are calculated for the CRSP standard deciles 1–10. The OLS betas and sum betas for the portfolios comprised of larger companies are approximately the same. As we move from Decile 1 (comprised of the largest companies) to Decile 10 (comprised of the smallest companies), sum betas become increasing larger than their OLS counterparts. For example, the OLS beta for decile 1 is 0.92, and the sum beta for decile 1 is also 0.92. The sum beta for decile 10, however (1.68), is significantly larger than the OLS beta for decile 10 (1.39).

All things held the same, the larger sum beta of decile 10 implies a smaller size premia (3.20%) than implied for its OLS beta counterpart (5.22%) (see Exhibit 4.7b). Sum betas tend to be larger for smaller companies than when using OLS betas. As a result, they tend to be less plagued by the overestimation problem due to incorrectly measuring beta.

Exhibit 4.7a: OLS Betas and Sum Betas, and their Respective Implied Size Premia, for CRSP NYSE/NYSE MKT/NASDAQ Deciles 1–10, as of December 31, 2018

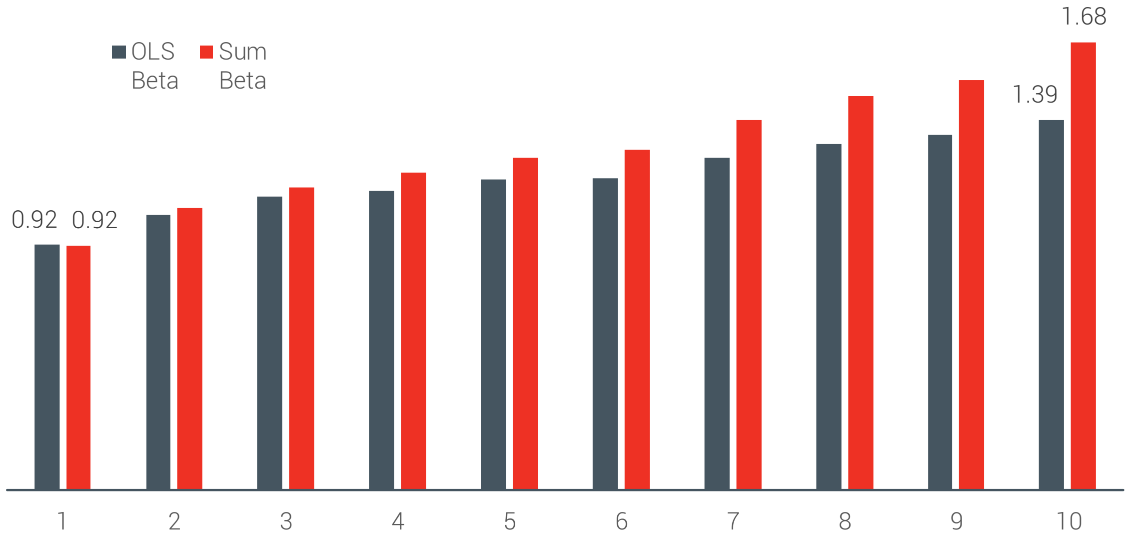
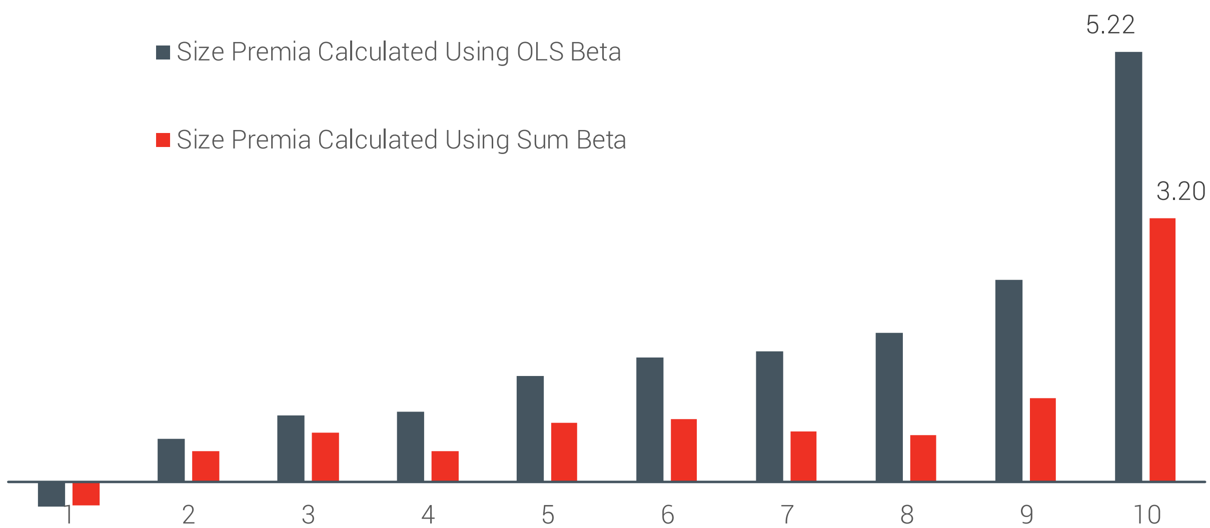


Exhibit 4.7b: Size Premia Calculated Using OLS Betas and Sum Betas, for CRSP NYSE/NYSE MKT/NASDAQ Deciles 1–10, as of December 31, 2018



Sources of underlying data for Exhibits 4.7a and 4.7b: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. (ii) Morningstar, Inc. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC. OLS and Sum betas are estimated from monthly return data in excess of the 30-day U.S. Treasury bill total return, January 1926–December 2018. Historical risk-free rate represented by the 93-year arithmetic mean income return component of 20-year U.S. government bonds (4.97%). Calculated in the context of the CAPM by multiplying the historical equity risk premium by beta. The historical equity risk premia in this example is estimated as the arithmetic annual mean return of the S&P 500 Index (11.88%) minus the arithmetic annual mean income return component of 20-year U.S. government bonds (4.97%) from 1926–2018.

In applying the capital asset pricing model (CAPM) (particularly for smaller businesses), we are looking for the most *accurate* estimate, and not the most *expedient* one. If you use an OLS beta for a small company by multiplying the OLS beta times the equity risk premium (ERP) estimate and adding an OLS-based size premium, you may not arrive at as accurate an estimate of the cost of equity capital as by multiplying a sum beta times the ERP estimate and adding a sum-beta-based size premium. You should be using the most accurate estimate of beta and the most accurate measure of the appropriate size premium. Having said that, whatever type of beta you ultimately choose to employ, you should match the source of the size premium (OLS or sum beta) with the type of beta estimate you have chosen for your subject company.

For example, for internal consistency, one should use a size premium derived using an OLS beta when the subject company beta is an OLS beta, and one should use a size premium derived using sum betas when the subject company beta is a sum beta (Exhibit 4.8).

Exhibit 4.8: Potential Impact on Cost of Equity Capital; Matching (or Mismatching) the Type of Beta Used in the CAPM Equation to the Type of Beta Used to Develop the Size Premium

Beta Used to Develop Size Premia	Beta Used in CAPM Equation: OLS Beta	Beta Used in CAPM Equation: Sum Beta
OLS Beta	A – even –	B Higher COE
Sum Beta	C Lower COE	D – even –

The resulting cost of equity capital resulting in the "matched" cases (Case A and Case D) do not necessarily have to equal (and likely will not), but they will tend to be within a reasonable range of each other. Using Cases B and C may lead to an incorrect estimate of cost equity capital. To be clear, we recommend using sum betas for the development of size premia, and to also use sum beta within the CAPM, (particularly if dealing with smaller companies), because sum betas tend to better explain the returns of smaller companies. However, in cases in which you do use OLS betas in CAPM, you should use an OLS beta derived size premium.

Data Issues

Critics of the size effect point out various issues with the data used, resulting in anomalies that people mistakenly have observed as the size effect. These data issues may include seasonality, bid/ask bounce bias, and delisting bias, among others.[4.10] In the following sections, we discuss the different compositions of portfolios in the CRSP Deciles Size Study data set and the Risk Premium Report Study data set.

Composition of the Smallest CRSP Deciles

We divided the CRSP 10th decile into subdeciles 10a and 10b (10a is the top half of the 10th decile, and 10b is the bottom half of the 10th decile) and further divided subdecile 10a into 10w and 10x, and subdecile 10b into 10y and 10z. This is the same breakdown of CRSP decile 10 that was previously presented in (i) the former Ibbotson Associates/Morningstar *SBBI® Valuation Yearbook,* and (ii) Duff & Phelps' *Valuation Handbook – U.S. Guide to Cost of Capital,* and now in the online Cost of Capital Navigator, which replaced the *Valuation Handbook – U.S. Guide to Cost of Capital* in 2018.

As of December 31, 2018, the reported size premium for the smallest 5% of companies by market capitalization as represented by CRSP subdecile 10b is 8.25%, and the size premium for the next smallest 5% of companies (as represented by CRSP subdecile 10a) is 3.71%, a difference of 4.54%.

What kind of companies populate subdeciles 10b and its top and bottom halves, 10y and 10z? The CRSP Deciles Size Study include all companies with no exclusion of speculative (e.g., start-up) or distressed companies whose market capitalization may be small because they are speculative or distressed. The inclusion of speculative or distressed companies in the database is one basis for criticism of the size effect. Exhibit 4.9 and Exhibit 4.10 display information about the types of companies that are included in decile 10y and decile 10z, respectively.[4.11]

[4.10] For a complete discussion of these issues, please refer to Pratt and Grabowski, op.cit.: Chapter 15A, "Other Data Issues Regarding the Size Effect".

[4.11] Exhibits 4.9 and 4.10 are as of September 2018 rather than December 2018 in order to mimic how the CRSP standard market-cap based portfolios are formed. The CRSP deciles portfolio compositions are reset quarterly (March, June, September, December), and then portfolio returns are calculated for these portfolio compositions over the *subsequent* quarter. As of December 2018, the most recent "reset" is September 2018.

Exhibit 4.9: Breakdown of Decile 10y Companies: Market Value of Equity between $109.462 and $184.785 million
September 30, 2018

Decile 10y	Market Value of Equity (in $millions)	Book Value of Equity (in $millions)	5-Year Average Net Income (in $millions)	Market Value of Invested Capital (in $millions)
95th Percentile	$180.567	$206.050	$14.660	$596.811
75th Percentile	164.136	120.009	6.538	222.403
50th Percentile	145.135	73.664	(2.419)	177.823
25th Percentile	124.566	32.859	(19.152)	142.236
5th Percentile	109.977	0.014	(44.510)	115.058

Decile 10y	Total Assets (in $millions)	5-Year Average EBITDA (in $millions)	Sales (in $millions)	Return on Book Equity (%)
95th Percentile	$1,480.151	$90.734	$936.174	34.2
75th Percentile	794.153	18.455	159.984	8.0
50th Percentile	163.197	-	49.969	(0.4)
25th Percentile	65.786	(12.043)	21.920	(54.7)
5th Percentile	23.418	(27.018)	0.480	(144.8)

Decile 10y	OLS Beta	Sum Beta
95th Percentile	2.26	2.56
75th Percentile	1.23	1.36
50th Percentile	0.55	0.73
25th Percentile	0.15	0.13
5th Percentile	(0.00)	(0.15)

Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. (ii) S&P *Capital IQ*. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

Exhibit 4.10: Breakdown of Decile 10z Companies: Market Value of Equity between $2.455 and $109.406 million
September 30, 2018

Decile 10z	Market Value of Equity (in $millions)	Book Value of Equity (in $millions)	5-Year Average Net Income (in $millions)	Market Value of Invested Capital (in $millions)
95th Percentile	$94.613	$115.874	$5.684	$225.088
75th Percentile	68.696	48.302	0.515	92.630
50th Percentile	41.957	21.530	(4.484)	56.026
25th Percentile	19.913	8.221	(13.786)	25.737
5th Percentile	8.086	(0.573)	(25.807)	9.623

Decile 10z	Total Assets (in $millions)	5-Year Average EBITDA (in $millions)	Sales (in $millions)	Return on Book Equity (%)
95th Percentile	$668.823	$23.478	$366.341	22.6
75th Percentile	114.147	3.654	67.537	3.3
50th Percentile	42.808	(1.188)	20.507	(14.1)
25th Percentile	17.667	(8.672)	2.466	(89.7)
5th Percentile	5.589	(19.331)	-	(181.1)

Decile 10z	OLS Beta	Sum Beta
95th Percentile	2.61	3.56
75th Percentile	1.31	1.82
50th Percentile	0.64	0.88
25th Percentile	0.16	0.27
5th Percentile	(0.00)	0.04

Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. (ii) S&P *Capital IQ*. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

From these data we can conclude:

- Betas used for calculating the size premium for subdecile 10y and subdecile 10z (using the OLS method of calculating betas) generally *understate* the beta, and therefore *overstate* the size premium. Note the small betas for companies in the 25th and 5th percentiles.

- Subdecile 10y and subdecile 10z are populated by many large (but highly leveraged) companies with small market capitalizations that probably do not match the characteristics of financially healthy but small companies (see "Total Assets", 95th percentile measures).

Stocks of the *troubled* companies included in the data probably are trading like call options (unlimited upside, limited downside). Even if you were to use the sum beta method, the beta estimates would likely be underestimated and the size premium overstated (see "Return on Book Equity", 25th percentile and 5th percentile).

Before using the size premium data for 10b or its top and bottom halves, 10y and 10z, the valuation analyst likely should determine if the mix of companies that comprise the subdeciles are indeed comparable to the subject company.

Composition of the Smallest Risk Premium Report Studies Portfolio

The Risk Premium Report Studies use a different methodology from the CRSP Deciles Size Studies. The Risk Premium Report Studies screen out speculative start-ups, distressed (i.e., bankrupt) companies, and other high-financial-risk companies. These studies measure beta using the sum beta method. This methodology was chosen to counter the criticism of the size effect by some that the size premium is a function of the high rates of return for speculative companies and distressed companies in the data set.

The Risk Premium Report Studies use the sum beta method to measure the size premium because it finds that betas of small companies in the data set (even after removing speculative, distressed, and other high-financial-risk companies) are underestimated if one uses the OLS method of estimating betas. Even after eliminating speculative, distressed, and other high-financial-risk companies and using the sum beta in measuring size, we still observe the size effect for a more recent period (since 1963).

The Risk Premium Report Study include a total of eight size measures, including six that are not based on market capitalization. Exhibit 4.11 shows the breakdown of companies in the Risk Premium Report Study in portfolio 25 (portfolio 25 is comprised of the smallest companies) for each of the eight size measures.

If the subject company is not highly levered, the companies in portfolio 25 may be more comparable to a small subject company, and therefore the size premium data for portfolio 25 may be more appropriate to use when dealing with very small companies.

Exhibit 4.11: Size Measure of Companies That Comprise Portfolio 25 of the Risk Premium Report Study
December 31, 2018

Portfolio 25	Market Value of Equity (in $millions)	Book Value of Equity (in $millions)	5-Year Average Net Income (in $millions)	Market Value of Invested Capital (in $millions)
Largest Company	$391.669	$170.954	$12.560	$457.602
95th Percentile	370.017	160.921	11.718	433.526
75th Percentile	245.900	123.411	9.328	288.874
50th Percentile	126.387	75.774	5.294	162.775
25th Percentile	65.274	37.854	2.424	71.848
5th Percentile	18.100	15.217	0.419	24.595
Smallest Company	8.801	7.738	0.051	11.282

Portfolio 25	Total Assets (in $millions)	5-Year Average EBITDA (in $millions)	Sales (in $millions)	Number of Employees
Largest Company	$333.066	$42.091	$278.924	725
95th Percentile	317.802	38.664	262.307	670
75th Percentile	242.495	26.200	176.122	448
50th Percentile	146.016	15.555	104.541	252
25th Percentile	61.984	6.677	46.176	115
5th Percentile	25.671	2.208	20.314	10
Smallest Company	13.058	0.258	4.648	4

Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. (ii) S&P *Research Insight*. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

Financial services companies (i.e., SIC code 6; those companies in finance, insurance, or real estate) are *excluded* from Risk Premium Report Study portfolios, primarily because some of the financial data used in the Risk Premium Report Study is difficult to apply to companies in the financial sector (e.g., "sales" at commercial banks). In addition, financial services companies tend to support a much higher ratio of debt-to-equity than do other industries, and so including them with non-financial firms may be an apples-to-oranges comparison that could lead to improperly skewed results. Moreover, companies in the financial services sector were poorly represented during the early years of the Standard & Poor's *Compustat* database.

Because companies in SIC code 6 are excluded from the set of companies used to perform the analyses presented in the Risk Premium Report, the data should not be used by an analyst estimating the cost of equity capital for a financial services company or other company in SIC code 6.

We also publish accounting-based fundamental risk information about the companies that comprise the 25 size-ranked portfolios for *each* of the eight size measures analyzed in the Risk Premium Report Study. This information includes:

- Five-year average operating income margin
- Coefficient of variation in operating income margin
- Coefficient of variation in return on book equity

The first statistic measures profitability, and the latter two statistics measure volatility of earnings.

This information provides the analyst with two important capabilities:

1. Additional tools to determine if the mix of companies that comprise the Risk Premium Report's portfolios are indeed comparable to the subject company.
2. The opportunity to gauge whether an increase (or decrease) adjustment to a risk premium or size premium (and thus cost of equity capital) is indicated, based on the company-specific differences of the subject company's fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived. (for more information, see the section entitled "Comparative Risk Study" in Chapter 10).

Has the Size Effect Disappeared in More Recent Periods?

Some research has suggested that in more recent years the size effect is greatly diminished, or has disappeared altogether. Often, 1981 is identified as the year after which the size effect has either diminished or disappeared. The primary reason for this is that in 1981 Banz examined the returns of NYSE small-cap companies compared to the returns of NYSE large-cap companies over the period 1926–1975, and found that there was a negative relationship between size–as measured by market capitalization–and return (i.e., as market capitalization *decreases*, returns *increase*). In effect, Banz is said to have "let the cat out of the bag" that small-cap companies offered greater returns, and that attracted more investment in small-cap companies. Prices were bid up, thus reducing overall returns for this asset class.

Hou and van Dijk posited that the apparent disappearance of the size effect after the early 1980s was due to cash flow shocks. Realized returns for small companies were generally less than expected because of negative cash flow shocks, and realized returns for large companies were generally greater than expected because of positive cash flow shocks.[4.12] What caused these unexpected cash flow shocks?

[4.12] Kewei Hou and Mathijs A. van Dijk, "Resurrecting the size effect: Firm size, profitability shocks, and expected stock returns", Ohio State University Fisher College of Business working paper, March 31, 2014. Copy available at: https://ssrn.com/abstract=1536804.

The number of newly public firms in the United States increased dramatically in the 1980s and 1990s compared with prior periods, and the profitability and survival rate of the newly public firms was generally less than the profitability and survival rates for firms that went public in previous years. After adjusting realized returns for the cash flow shocks, the result was that returns of small firms on a pro forma basis exceeded the returns of large firms by approximately 10% per annum, consistent with the size premium in prior periods.

A more direct reason often cited for a diminished size effect in more recent years was possibly most succinctly stated by Horowitz, Loughran, and Savin, who suggested that "it is quite possible that as investors became aware of the size effect, small firm prices increased (thus lowering subsequent returns)".[4.13] This conjecture may be supported by the sheer number of small-cap companies that have come into existence since Banz's 1981 article that demonstrated that small-cap companies exhibited significantly greater performance over the period from 1926 to 1975. [4.14]

In a more recent study, the authors found the size effect exists and is statistically significant when one accounts for quality differences among companies. They found that a key variable in explaining the changing size effect over time is the markets pricing of firm quality (as measured by profitability, stability, growth and safety) versus junk. They find that this relationship has a far stronger explanatory power than other factors (relationship of size to the market, value, or momentum). This finding holds whether size is measured by market capitalization or non-market based ("fundamental") measures. Further, this finding holds for each of the 30 industries and 23 countries studied. Further, they found that the size effect holds in periods where other researchers have claimed the size effect has disappeared. The authors also found that the size effect holds not only during the month of January (the "January effect") but through other months as well.[4.15]

In another recent study the author finds that when one examines established (i.e., companies that are not start-up), profitable companies and not financially distressed, there is strong evidence supporting the size effect including in periods where other researchers have claimed the size effect has disappeared.[4.16]

Size Effect: The Big Picture On Small versus Large

We performed analyses to investigate which of two hypothetical investors would have ended up with more money in their pocket over various holding periods within the full range of monthly CRSP decile data (January 1926–December 2018):

- "Investor A" invests only in large-cap companies
- "Investor B" only invests in small-cap companies.

4.13 Joel L. Horowitz, Tim Loughran, and N.E. Savin, "The disappearing size effect", *Research in Economics* (2000), page 98.

4.14 Banz, Rolf W. "The Relationship between Return and Market Value of Common Stocks". *Journal of Financial Economics* (March 1981): 3–18. Professor Banz's 1981 article is often cited as the first comprehensive study of the size effect.

4.15 Asness, Clifford S., Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz, and Lasse Heje Pedersen, "Size Matters, If You Control Your Junk," Journal of Financial Economics 129 (2018): 479-509.

4.16 Grabowski, Roger J., "The Size Effect Continues to Be Relevant When Estimating the Cost of Capital," Business Valuation Review 37 (3) (2018).

To do this, we first calculated the terminal index value of $1.00 invested for every possible combination of monthly start-dates and end-dates for CRSP decile 1 (comprised of the largest-cap companies) and CRSP decile 10 (comprised of the smallest-cap companies) over the January 1926 to December 2018 period.[4.17] The total number of monthly start-dates and end-dates combinations between January 1926 and December 2018 is 623,286.

We then subtracted the terminal index value of large-cap companies from the terminal index value of small-cap companies for *each* of the 623,286 start-date/end-date combinations. If the terminal index value of small-cap companies was *greater* than the terminal index value of large-cap companies, this would indicate small-cap companies earned a *higher* return over that period for the investor.

Example: $1.00 invested in large-cap companies from January 1926 would have grown to $3,951.18 by the end of December 2018. Alternatively, $1.00 invested in small-cap companies from January 1926 would have grown to $82,627.79 by the end of December 2018. Investing in small-cap companies would have resulted in $78,676.61 ($82,627.79 – $3,951.18) *more* money in your pocket than investing in large-cap companies over this period.

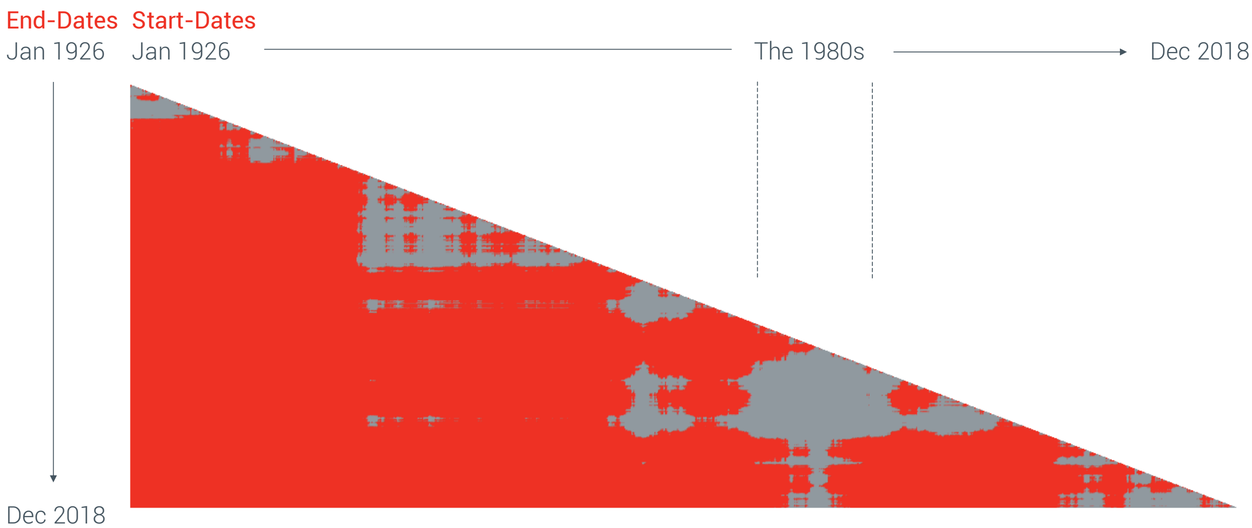
These calculations were performed for *every* possible monthly start-date and end-date combination between January 1926 and December 2018. The result of this analysis was that small-cap companies outperformed large-cap companies in 526,027 of the cases (84.4%), and large-cap companies outperformed small-cap companies in 97,259 cases (15.6%).

These results are shown in Exhibit 4.12, where the difference in the terminal index value between small-cap companies and large-cap companies for all 623,286 possible start-date/end-date combinations from January 1926 to December 2018 are mapped. In Exhibit 4.12, if the terminal index value for small-cap companies is *greater* than the terminal index value for large-cap companies over a start-date/end-date combination (i.e., small-cap companies outperformed large-cap companies over that period), it is shown in red (526,027 cases). Alternatively, if the terminal index value for small-cap companies is *less* than the terminal index value for large-cap companies over a start-date/end-date combination (i.e., large-cap companies outperformed small-cap companies over that period), it is shown in gray (97,259 cases).

The significance of the large gray area in Exhibit 4.12 under start-dates that begin in the 1980s will be discussed in more detail later in this chapter.

[4.17] The terminal index value in all cases presented here is the amount that $1 invested on the start-date would have grown to (or decreased to) as of the end-date. All terminal index values in this section are calculated geometrically.

Exhibit 4.12: CRSP Decile 10 (small-cap companies) Terminal Index Values *Minus* CRSP Decile 1 (large-cap companies) Terminal Index Values for 623,286 Start-Date/End-Date Combinations; Red = Small-Cap Companies Outperformed Large-Cap Companies Over the Period, Gray = Large-Cap Companies Outperformed Small-Cap Companies Over the Period
January 1926–December 2018



Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. Large-cap companies and small-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ deciles 1 and 10, respectively. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

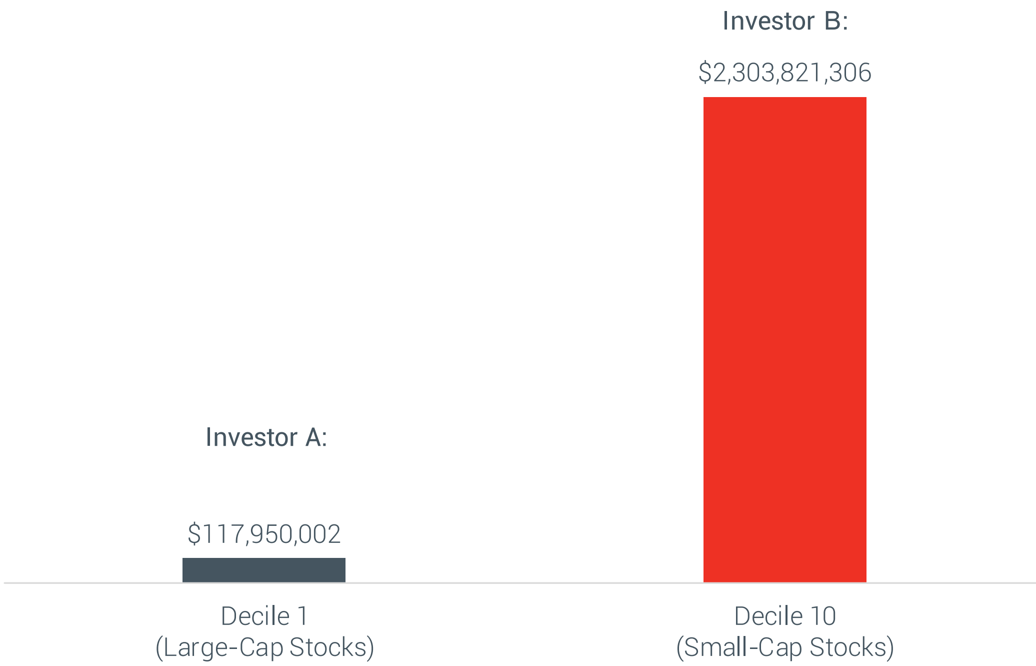
The results in Exhibit 4.12 are merely a record of whether small-cap companies outperformed large-cap companies, or vice versa, over the 623,286 possible start-date/end-date periods, with no regard to the *magnitude* of the outperformance. The "magnitude" of overperformance can be illustrated with the following example.

If hypothetical Investor A, who invests only in CRSP Decile 1 (comprised of the *largest* companies), had invested $1 in *each* of the 623,286 possible start-date/end-date investment horizons between January 1926 and December 2018, her $623,286 total investment would have grown to $117,950,002 (i.e., $118.0 million, see Exhibit 4.13).

Alternatively, if hypothetical Investor B, who invests only in CRSP Decile 10 (comprised of the *smallest* companies), had invested $1 in each of the 623,286 possible start-date/end-date investment horizons between January 1926 and December 2018, his $623,286 total investment would have grown to $2,303,821,306 (i.e., $2.3 trillion).

Investor B, who invested only in small companies, ends up with 19.5 times as much money in his pocket ($2,303,821,306 ÷ $117,950,002) than Investor B, who only invests in large companies.

Exhibit 4.13: Proceeds From an Investment of $1 in *Each* of the 623,286 Possible Start-Date/End-Date Investment Horizons Between January 1926 and December 2018; "Investor A" invests only in large-cap stocks, "Investor B" invests only in small-cap stocks



Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. Large-cap companies and small-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ deciles 1 and 10, respectively. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

Size Effect: A Closer Examination

In Exhibit 4.14, a more detailed summary of these results is shown, where the holding periods are limited to *exactly* 1 month, 5-years, 10-years, 20-years, and 30-years, instead of all 623,286 possible start-date and end-date combinations. The entire January 1926–December 2018 period is examined, as well as three more recent start date windows: April 1981–December 2018, January 1990–December 2018, and January 2000–December 2018. All three of these three more recent periods are *after* Banz wrote his March 1981 article that identified the size effect, and so they are labeled "Post Banz".

In Exhibit 4.14 the number of periods examined is shown first, followed by the outperformance percentage of the total periods in parentheses.

Exhibit 4.14: Small-cap Companies' Performance minus Large-cap Companies' Performance Over Periods of Exactly 1, 60, 120, 240, and 360 Months
January 1926–December 2018

Holding Period	All Dates Jan 1926–Dec 2018	Post Banz Apr 1981–Dec 2018	Post Banz Jan 1990–Dec 2018	Post Banz Jan 2000–Dec 2018
Exactly 1 month				
Small Stocks Outperform	526 (47%)	208 (46%)	169 (49%)	118 (52%)
Large Stocks Outperform	590 (53%)	245 (54%)	179 (51%)	110 (48%)
Exactly 60 months (5 years)				
Small Stocks Outperform	591 (56%)	177 (45%)	172 (60%)	108 (64%)
Large Stocks Outperform	466 (44%)	217 (55%)	117 (40%)	61 (36%)
Exactly 120 months (10 years)				
Small Stocks Outperform	697 (70%)	184 (55%)	184 (80%)	85 (78%)
Large Stocks Outperform	300 (30%)	150 (45%)	45 (20%)	24 (22%)
Exactly 240 months (20 years)				
Small Stocks Outperform	718 (82%)	167 (78%)	109 (100%)	–
Large Stocks Outperform	159 (18%)	47 (22%)	0 (0%)	–
Exactly 360 months (30 years)				
Small Stocks Outperform	688 (91%)	79 (84%)	–	–
Large Stocks Outperform	69 (9%)	15 (16%)	–	–

Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. Large-cap companies and small-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ deciles 1 and 10, respectively. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

In the top row of Exhibit 4.14 (in which the holding period is restricted to a single month), large-cap companies outperformed small-cap companies in the January 1926–December 2018 period (53%), and in the "Post-Banz" April 1981–December 2018 and January 1990–December 2018 time horizons (54% and 51%, respectively). In the more recent January 2000–December 2018 time horizon small-cap companies outperformed 52% of the time.

As the holding period is increased, and the time that small-cap companies and large-cap companies are given to "race" against each other is lengthened, small-cap stocks tend to *increasingly* outperform large-cap stocks. For example, over the entire range January 1926–December 2018 (see leftmost column of Exhibit 4.14), as the holding period is increased to 60 months (5-years), to 120 months (10-years), to 240 months (20-years) and finally to 360 months (30-years), small stocks increasingly outperform large stocks (56%, 70%, 82%, and 91% of the time, respectively).

This same pattern of *increasing* outperformance of small stocks as the holding period is *increased* can also be seen in the three "Post Banz" periods.

The 1980s and the Size Effect

To examine the significance of the large gray area under start-dates that begin in the 1980s previously alluded to in Exhibit 4.12, we performed the following analysis:

1. All possible and *identical* "240-month x 240-month" sized wedges that exist in the larger "wedge" shown in Exhibit 4.12 were identified. Over the time period January 1926–December 2018, the number of possible and identical 240-month x 240-month sized wedges in Exhibit 4.12 is 877.[4.18]

2. We calculated the proceeds from our hypothetical **Investor A** investing $1 in *each* of the 28,920 possible and identical start-date/end-date investment horizons in *each* of the "240-month x 240-month" sized wedges (Investor A invests only in CRSP Decile 1, which is comprised of the largest companies).

3. We calculated the proceeds from our hypothetical **Investor B** investing $1 in *each* of the 28,920 possible and identical start-date/end-date investment horizons in *each* of the "240-month x 240-month" sized wedges (Investor B invests only in CRSP Decile 10, which is comprised of the smallest companies).

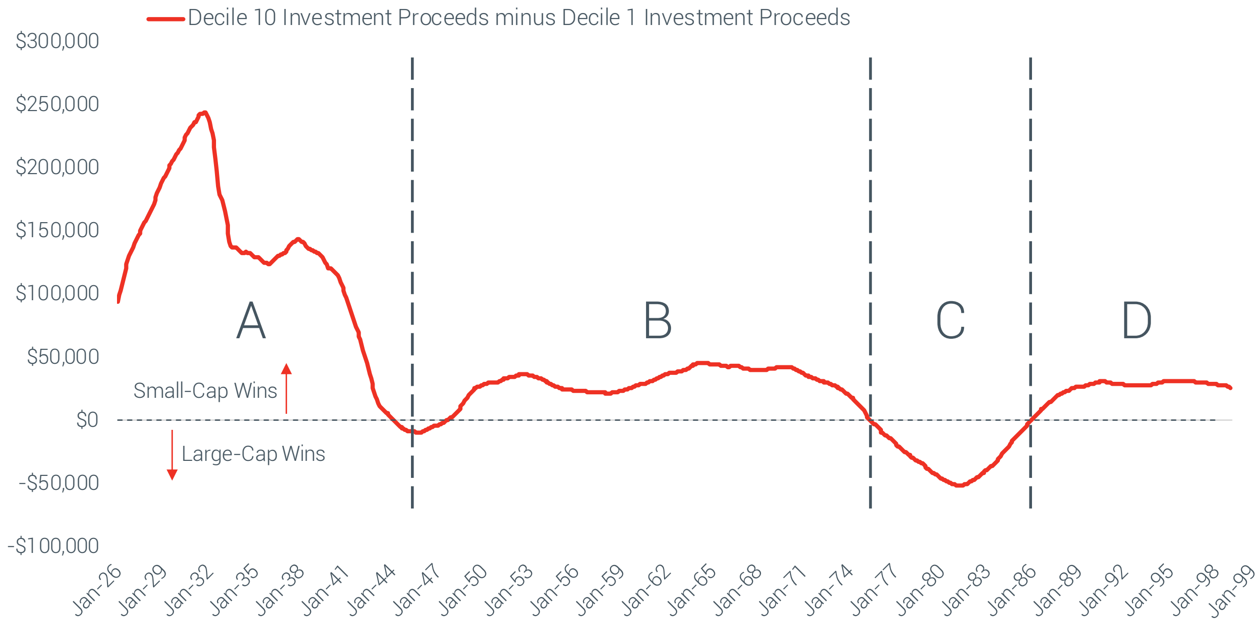
4. Finally, for each of the 877 "240-month x 240-month" sized wedges, Investor A's "large-cap company" investment proceeds were subtracted from Investor B's "small-cap company" investment proceeds.

The results of this analysis are shown in Exhibit 4.15. (Next Page)

[4.18] By "identical", we mean (i) each wedge is exactly 240 months x 240 months (20 years) in size, and (ii) the possible start-date/end-date combinations within each of the 877 "240-month x 240-month wedges" are identical in number (28,920), and (iii) each of the 28,920 possible start-date/end-date combinations within each of the 877 "240-month x 240-month wedges" has an *exact* equivalent possible start-date/end-date combination in each of the other 877 "240-month x 240-month wedges". Thus, for each of the 877 wedges, the number of periods measured and the length of those periods is exactly identical to the number of periods and length of periods in each of the other 877 wedges.

Exhibit 4.15: Investor A's "large-cap company" Investment Proceeds Subtracted from Investor B's "small-cap company" Investment Proceeds for Each Possible and Identical "240-month x 240-month" Sized Wedge from January 1926–December 2018

Note: *January 1999 is the last "start month" for which a "240-month x 240-month" sized wedge could be calculated ending Dec. 31, 2018.*



Source of underlying data: CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2019 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. Large-cap companies and small-cap companies are represented by CRSP NYSE/NYSE MKT/NASDAQ deciles 1 and 10, respectively. Used with permission. All rights reserved. Calculations performed by Duff & Phelps, LLC.

Each of the 877 points that comprise the solid red line in Exhibit 4.15: (i) is made up of the results of 28,920 separate investments of $1 in *each* of 28,920 start-date/end-date time periods in the given "240-month x 240-month" wedge being examined, and (ii) is directly comparable to every other point in the graph. In other words, there are a lot of observations in Exhibit 4.15, and those observations are all comparable to each other in an "apples to apples" fashion.

In Exhibit 4.15, if the investment proceeds of investing in small-cap companies are *greater* than the investment proceeds of investing in large-cap companies, the red line is *above* the dashed horizontal "$0" line. Alternatively, if the investment proceeds of investing in small-cap companies are *less* than the investment proceeds of investing in large-cap companies, the red line is *below* the dashed horizontal "$0" line.

There are at least four observations about the results shown in Exhibit 4.15:

- **Observation 1:** Small-cap companies usually win. Investor B's "small-cap company" investment proceeds were *greater* than Investor A's "large-cap company" investment proceeds in 705 (80.4%) of the 877 identical "240-month x 240-month" wedges examined.

- **Observation 2:** Small-cap companies outperformed large-cap companies to a greater degree in *earlier* periods (see area "A" in Exhibit 4.15)[4.19] than they did in *later* periods (see areas "B", "C", and "D").

- **Observation 3:** Small-cap companies performed *poorly* relative to large-cap companies in the "240-month x 240-month" wedges that overlap the 1980s (see area "C" in Exhibit 4.15).

- **Observation 4:** As soon as the influence from the 1980s is in the rear-view mirror, small-cap companies seem to regain their footing, and the size effect in area "D" seems to return to what it was in area "B".[4.20]

Controlling for Small-Cap Companies' Significant Outperformance of Large-Cap Companies in Earlier Periods

This section started with the question of whether the size effect has disappeared in more recent periods. The empirical evidence presented thus far suggests that the size premia is likely alive and well, even in the periods following the 1981 publication of Rolf Banz's seminal article.[4.21] However, the evidence also suggests that the size effect may be of *diminished* strength in more recent years, especially when compared to very early periods.

For example, one of the four observations about the results in Exhibit 4.15 was that small-cap companies outperformed large-cap companies to a greater degree in the *earlier* periods of 1926–1945 (see area "A" in Exhibit 4.15) than they did in *later* periods. One might reasonably reckon that "most" of the size effect over the 1926–2018 time horizon happened in the earlier years, as represented by the 20-year period from 1926–1945 (see area "A" in Exhibit 4.15), and that if these early years were controlled for (i.e., "excluded ") in the calculations of size premia, that the size premia might be severely weakened, or disappear altogether.

We tested to see what would happen if the first 20 years (1926–1945), a period during which the size effect was stronger than it was in later periods, were *excluded* from the calculations of 2018 year-end size premia. In Exhibit 4.16, the results of this analysis are shown. The solid red line in Exhibit 4.16 is the size premium for CRSP Decile 10, as of December 31, 2018, calculated as if the CRSP data started in *each* year from 1926–2018 (instead of just 1926).

[4.19] Area "A" represents the first 20 years of Exhibit 4.15 (i.e., 1926–1945). "1926–1945" was arbitrarily selected to represent the earlier years in Exhibit 4.15. For example, 1926–1944 (or even 1943) could just as easily have been selected; 1926–1945 was selected because it is a round 20-year period.

[4.20] Dimson, Marsh and Staunton address this in a recent paper: "Over the period 1984–1997, the small-cap premium turned negative; although, ironically, after we highlighted the demise of the size effect, U.S. small caps performed very well over the first decade of the 21st century in both relative and absolute terms." See: Elroy Dimson, Paul Marsh and Mike Staunton, *The Journal of Portfolio Management* Special QES Issue 2017, 43 (5) 15-37; DOI: https://doi.org/10.3905/jpm.2017.43.5.015.

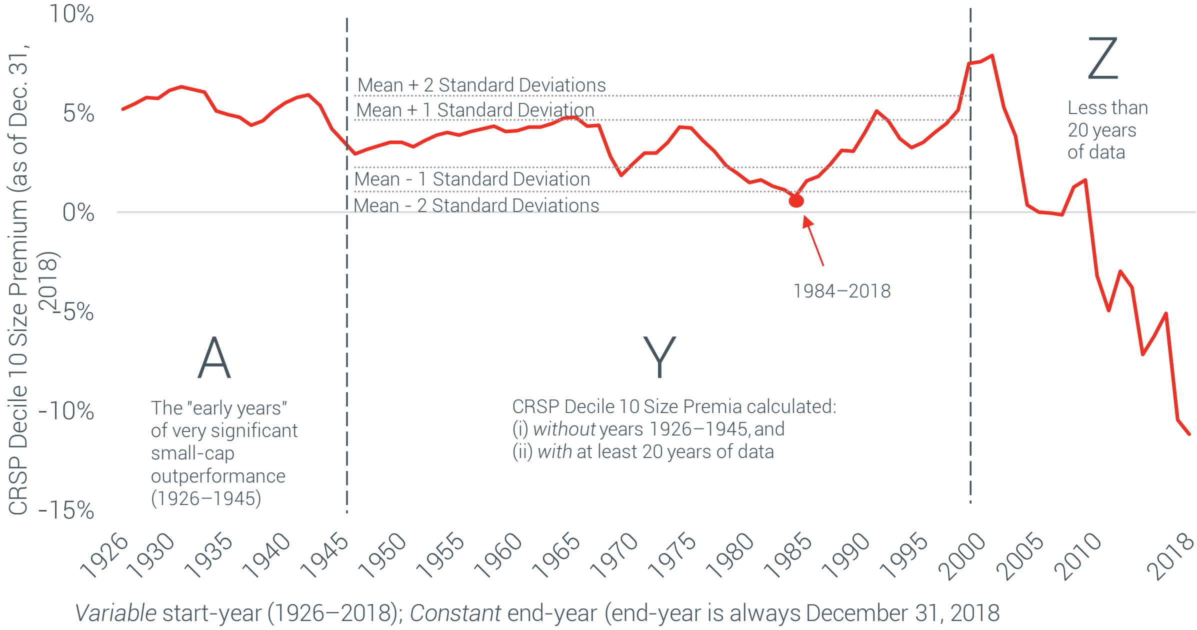
[4.21] Banz, Rolf W. "The Relationship between Return and Market Value of Common Stocks". *Journal of Financial Economics* (March 1981): 3–18. Professor Banz's 1981 article is often cited as the first comprehensive study of the size effect.

For example, the leftmost point in Exhibit 4.16 is the size premium for CRSP Decile 10 calculated over the time period 1926–2018 (93 years). The second-most leftmost point in is the size premium for CRSP Decile 10 as of December 31, 2018 calculated over the time period 1927–2018 (92 years), the third-most leftmost point is the size premium for CRSP Decile 10 as of December 31, 2018 calculated over the time period 1928–2018 (91 years), etc., etc., until the *rightmost* point in Exhibit 4.16 is the size premium for Decile 10 as of December 31, 2018 calculated over the time period 2018–2018 (1 year).

Area "A" in Exhibit 4.16 is the equivalent of area "A" from Exhibit 4.15. Area "A" in both exhibits is represented by the "early years" of 1926–1945, during which small-cap companies' outperformance of large-cap was significantly greater than it was in later periods. In area "A" in Exhibit 4.16, the year-end 2018 CRSP Decile 10 size premia is calculated with start-years of 1926–1945, and a *constant* end-year of 2018.

Area "Z" of Exhibit 4.16 is the year-end 2018 CRSP Decile 10 size premia as of December 31, 2018 calculated with start-years of 1999–2018, and a *constant* end-year of 2018. Each of the calculations in area Z includes less than 20 years of data, and is therefore excluded from any further analysis because of the short time horizon over which they are calculated.

Exhibit 4.16: CRSP Decile 10 Size Premium Calculated Through 2018 (in each case), and *Different* Start-Years (1926–2018).



Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2018 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. To learn more about CRSP, visit www.crsp.com. (ii) Morningstar, Inc. Used with permission. All rights reserved. Calculations by Duff & Phelps.

The real area of interest in Exhibit 4.16 is area "Y". In area "Y" the year-end 2018 CRSP Decile 10 size premia is calculated with start-years of 1946–1998, and a *constant* end-year of 2018. Note that the 93 points that comprise the red line in Exhibit 4.16 are *not* "apples to apples" comparable, because the time horizon over which each point is calculated is *different* (93 years, 92 years, 91 years,...etc.). However, the size premia in area "Y" are each calculated with *at least* 20 years of data.[4.22]

All of the size premia in area "Y" are also calculated *without* any data from 1926–1945, the area "A" years in which small-cap companies' outperformance of large-cap companies was significantly greater than it was in later periods. In other words, the huge small-cap outperformance of the 1926–1945 period has been "controlled for" (i.e., excluded) in all size premia calculations in area "Y".

The resulting CRSP Decile 10 size premia calculated area "Y" are all positive, even after controlling for the huge small-cap outperformance of the "early years" in area "A". As a matter of fact, all but one data point (i.e., the year-end 2018 CRSP Decile 10 size premia calculated using data from 1984–2018) within area "Y", had a calculated size premium higher than the mean (i.e., average) minus two standard deviations. In other words, with the one exception noted, all the size premium observations calculated in area "Y" were in excess of the lower-bound 95% confidence interval

4.22 The leftmost point in Area Y in Exhibit 4.16 is the CRSP Decile 10 size premium calculated using data from 1946–2018 (73 years); the rightmost point in Area Y in Exhibit 4.16 is the CRSP Decile 10 size premium calculated using data from 1999–2018 (20 years). The next calculation (1999–2017) has less than 20 years of data (19 years) and therefore falls into area "Z". Area "Z" results are excluded from any further analysis in this section because of the short time horizon over which they are calculated.

(mean minus two standard deviations), with the indicated size premia always being positive (greater than 0.0%).

These analyses suggest:

- The size effect is cyclical. Sometimes small-cap companies outperform large-cap companies, and sometimes large-cap companies outperform small-cap companies.[4.23]
- The longer the holding period over which small-cap companies and large-cap companies are given to "race" against each other, the more likely it is that small-cap companies will outperform large-cap companies.[4.24]
- This implies that over the longer-term (which is the default period over which most business valuations are done), the size effect is indeed a significant factor that should likely be accounted for in the development of cost of capital estimates.
- The 1980s were not kind to small capitalization stocks. During this period, the size effect likely was on a cyclical low, or even significantly negative.
- After the influence from the 1980s is in the rear-view mirror, small-cap companies seem to regain their footing, and the size effect seems to return to levels similar to those in the decades preceding the 1980s.
- The evidence suggests that the size effect has diminished in strength in more recent years, especially when compared to the "early years" 1926–1945, during which small-cap companies outperformed large-cap companies by a large magnitude.
- The size effect is still significant even *after* controlling for the huge small-cap outperformance of the "early years" 1926–1945.

Relationship of Size and Liquidity

Liquidity affects the cost of capital. For this purpose, *liquidity* refers to the speed at which a large quantity of a security can be traded with a minimal impact on the price and at the lowest cost. Banz's 1981 musing as to whether "...size per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size" may have been cannily prescient. Research on returns as related to "size" is abundant, but over time a growing body of work investigating the impact of "liquidity" on returns has emerged.

Capital market theory also assumes liquidity of investments. Many of the observations about risk and return are drawn from information for liquid investments. Investors desire liquidity and require

4.23 See: Roger J. Grabowski, "The Size Effect – It Is Still Relevant", *Business Valuation Review*, Volume 35, Number 2, Summer 2016.

4.24 Empirically, estimation error of premiums goes down with the square root of time, unlike beta or standard deviation estimation error which goes down by the square root of the number of observations. Thus, there can be long periods of negative results for positive premiums. See also: Fama, Eugene F. and French, Kenneth R., "Long-Horizon Returns" (November 20, 2017). Chicago Booth Research Paper No. 17-17; Fama-Miller Working Paper. Available at: SSRN: https://ssrn.com/abstract=2973516 or http://dx.doi.org/10.2139/ssrn.2973516 and Fama, Eugene F. and French, Kenneth R., "Volatility Lessons" (November 1, 2017). Chicago Booth Research Paper No. 17-33; Fama-Miller Working Paper. Available at: SSRN: https://ssrn.com/abstract=3081101.

greater returns for illiquidity. But the degree of liquidity is one of the risk factors for all investments. Any discussion of a liquidity premium, therefore, would be incomplete without accounting for underlying stock risks before considering relative liquidity.

Stocks of small companies generally do not have the same level of liquidity as large-company stocks. This is likely a function of the mix of shareholders and underlying risk characteristics. Many institutional investors do not own stocks in small companies because they have too much money to invest. Were they to invest as little as 1% of their available funds in a small company, they would be likely to control the company. Institutional investors generally want liquidity to move into and out of positions in a single firm. Therefore, one does not see the breadth of investors investing in small-company stocks.

Further, small companies are followed by only a small window of analysts, if at all. This makes it more difficult for investors to evaluate small firms.

Is the size premium simply the result of differences in liquidity? If one is valuing a small business, that business, if it were publicly traded, would likely never have the same breadth of shareholders as a large publicly traded company, and whatever impact the relative illiquidity of small companies has on the cost of capital will carry over to any small business.

Some analysts have suggested that the size effect should be set aside because various studies have ignored transaction costs in measuring rates of return. The analysts point out that small stocks often have higher transaction costs than large stocks. In addition, the historical size premium can be greatly reduced if one makes certain assumptions about transaction costs and holding periods. However, in applying the income approach to valuation, analysts typically use projected net cash flows that do not make any adjustment for an investor's hypothetical transaction costs. It may be that small stocks are priced in a way that increases the rates of return so as to reward investors for the costs of executing a transaction. If so, it would be a distortion to express the discount rate on a net-of-transaction-cost basis while the net cash flow projections are on a before-transaction-cost basis.

Academic studies support the hypotheses that illiquidity is a factor in pricing and returns of stocks and that returns of small firms are more sensitive to market liquidity. Moreover, any reasonable adjustment for transaction costs should recognize that investors can mitigate these costs on an annual basis by holding their stocks for a longer period. In fact, investors in small companies tend to have longer holding periods than investors in large companies.

First, let's examine some of the research.

As early as 1986, Amihud and Mendelson, demonstrated that "…market-observed average returns are an increasing function of the spread…" (i.e., less liquid stocks, as measured by a larger bid-ask spread, outperform more liquid stocks), and further concluded that the "…higher yields required on higher-spread stocks give firms an incentive to increase the liquidity of their securities, thus reducing their opportunity cost of capital".[4.25]

[4.25] Amihud, Yakov and Haim Mendelson, 1986, "Asset Pricing and the Bid-Ask Spread", *Journal of Financial Economics* 17, 223–249.

In a 2013 article, Ibbotson, Chen, Kim, and Hu suggested that while the typical measures of liquidity employed in the literature are each "...highly correlated with company size", they demonstrate that liquidity, as measured by annual stock turnover, "...is an economically significant investment style that is just as strong, but distinct from traditional investment styles such as size, value/growth, and momentum".[4.26] Analyzing the performance of a broad universe of U.S. stocks from 1972–2011, the authors go on to say that "...there is an incremental return from investing in less liquid stocks even after adjusting for the market, size, value/growth, and momentum factors", and conclude that "...equity liquidity is the missing equity style".

The authors identify two main sources of the greater returns of less liquid stocks. The first is that "investors like liquidity and dislike illiquidity", and "...a premium has to be paid for any characteristic that investors demand, and a discount must be given for any characteristic investors seek to avoid". Thus, "...the investor in less liquid stocks gets lower valuations, effectively buying stocks at a discount".

As we discussed in Chapter 2, one can think of risk in terms of popularity. For example, illiquidity is typically considered a risk, and less liquid stocks are considered less popular. One can classify less liquid stocks as less popular than brand name stocks that are in the news, having more analyst coverage and greater trading volume. Similarly, the size premium can be thought of as a risk measure that encompasses both illiquidity risk and underlying business risk; small capitalization stocks are typically less popular.

In a 2018 update to the 2013 article, Ibbotson and Kim examine market data from 1972–2017 and conclude that liquidity, as measured by stock turnover, meets the four criteria that characterize a benchmark investment style that William F. Sharpe defined in a 1992 article: (i) "identifiable before the fact", (ii) "not easily beaten", (iii) "a viable alternative", and (iv) "low in cost":[4.27, 4.28]

Identifiable Before the Fact: Given that Ibbotson and Kim's measure of liquidity was the previous year's turnover of the stock, the liquidity measure used is (by definition) "*identifiable before the fact*".[4.29]

Not Easily Beaten: Ibbotson and Kim then compared the 1st quartile returns of the various styles, and these all outperformed the equally weighted market portfolio. The returns from the low liquidity quartile were comparable to the other styles, beating size and momentum, but trailing value. They consider all four styles to be "*not easily beaten*".

4.26 See Roger G. Ibbotson, Zhiwu Chen, Daniel Y.-J. Kim, and Wendy Y. Hu, "Liquidity as an Investment Style", *Financial Analysts Journal* Vol. 69(3): 30–44, May/June 2013. Copy available at www.zebracapm.com.

4.27 The "2018 update to the 2013 article" is Roger G. Ibbotson and Daniel Y.-J.Kim, "Liquidity as an Investment Style, 2018 Update", February 13, 2018. The section on the 2018 update herein is largely excerpted from Roger G. Ibbotson and Daniel Y.-J.Kim's writing in same. Copies of the 2018 update are available at www.zebracapm.com. Roger Ibbotson is Professor Emeritus of Finance, Yale School of Management, and Chairman, Zebra Capital Management, LLC. Daniel Y.-J.Kim is Director of Research, Zebra Capital Management, LLC.

4.28 Sharpe, William F., 1992, "Asset Allocation: Management Style and Performance Measurement". *Journal of Portfolio Management*, Vol. 18, No. 2 (Winter):7–19.

4.29 Other liquidity measures could have met that criteria as well, but Ibbotson and Kim chose turnover because it was simple, easy to measure, and has a significant impact on returns.

A Viable Alternative: Ibbotson and Kim examined double sort portfolios comparing liquidity with size, value, and momentum in four-by-four matrices. The impact of liquidity on returns was somewhat stronger than size and momentum, and roughly comparable to value. It was also additive to each style. Thus they determined that liquidity was "*a viable alternative*" to size, value, and momentum.

Low in Cost: Ibbotson and Kim demonstrated that less liquid portfolios could be formed "*at low cost*". The portfolios they examined were formed only once per year, and 64.27% of the stocks stayed in the same quartile. The high-performing low quartile had 78.55% of the stocks stay in that quartile. Thus the liquidity portfolios themselves exhibit low turnover, which can keep their costs low.

Ibbotson and Kim demonstrate that liquidity is "a viable alternative" to each of the three other well established styles (size, value/growth, and momentum) by focusing on distinguishing turnover from size, value, and momentum by constructing "double-sort" quartile portfolios that combine liquidity with each of the other styles (in turn). In each of these analyses, the "liquidity effect" held regardless of size, value/growth, and momentum groupings.

For example, it is often presumed that investing in less liquid stocks is equivalent to investing in small-cap stocks. To determine if liquidity is effectively a proxy for size, they constructed equally weighted double-sort portfolios in capitalization and turnover quartiles. Exhibit 4.17 reports the annualized geometric mean (compound) return, arithmetic mean return, and standard deviation of returns along with the average number of stocks in each intersection portfolio.

Exhibit 4.17: Summary Statistics of Size and Liquidity "Double Sort" Quartile Portfolios 1972–2018

	Low Liquidity	Mid-Low Liquidity	Mid-High Liquidity	High Liquidity	Liquidity Effect (%)
Micro-Cap					
Geometric Mean (%)	15.45	14.97	9.08	-0.72	**16.17**
Arithmetic Mean (%)	17.80	18.53	14.22	4.43	
Standard Deviation (%)	22.78	28.61	34.37	33.16	
Avg. Number of Stocks	347	181	123	98	
Small-Cap					
Geometric Mean (%)	15.19	13.95	11.72	5.45	**9.74**
Arithmetic Mean (%)	16.83	16.44	14.96	9.53	
Standard Deviation (%)	19.40	23.63	26.84	30.02	
Avg. Number of Stocks	199	201	173	174	
Mid-Cap					
Geometric Mean (%)	13.56	13.33	12.46	7.84	**5.72**
Arithmetic Mean (%)	14.92	15.01	14.55	11.31	
Standard Deviation (%)	17.68	19.60	21.52	27.33	
Avg. Number of Stocks	129	178	203	239	
Large-Cap					
Geometric Mean (%)	11.12	11.96	11.46	8.62	**2.50**
Arithmetic Mean (%)	12.33	13.07	12.97	11.51	
Standard Deviation (%)	16.20	15.39	17.74	24.48	
Avg. Number of Stocks	73	189	249	237	
Size Effect (%)	**4.33**	**3.01**	**-2.38**	**-9.34**	

Source: Compound annual returns (%) from 1972–2018. Calculated by Zebra Capital Management at www.zebracapm.com. This is an update to the research published in Ibbotson, Roger G., and Daniel Y.-J Kim, "Liquidity as an Investment Style: 2018 Update," available at www.zebracapm.com. Updated version of: Ibbotson, Roger G., Chen, Zhiwu, Kim, Daniel Y.-J., and Hu, Wendy Y. "Liquidity as an Investment Style," *Financial Analysts Journal*, May/June 2013, updated with 2013–2017 data.

Across the micro-cap quartile in Exhibit 4.17, the low-liquidity portfolio earned a geometric mean return of 15.45% per year in contrast to the high-liquidity portfolio returning –.72% per year, suggesting that the liquidity effect is the *strongest* (16.17%) among micro-cap stocks, and then declines from small- to mid- to large-cap stocks. Note that the micro-caps row contains both the *highest* return and the *lowest* returns. 4.32

Across the large-cap quartile, the low- and high-liquidity portfolios returned 11.12% and 8.62% 4.33
respectively, producing a liquidity effect of 2.50%. 4.34

Within the two mid-size portfolios, the liquidity return spread is also significant. Therefore, size does not capture liquidity (i.e., the liquidity premium *holds* regardless of size group). Conversely, the size effect does *not* hold across all liquidity quartiles, especially in the highest turnover quartile (-9.34%).

A "heat map" of the size and liquidity "double sort" quartile portfolios is presented in Exhibit 4.18. In Exhibit 4.18, the deeper the red, the *higher* the return, and the darker the gray, the *lower* the return. For example, the *highest* return over the 1972–2018 period was produced by low-liquidity/micro-cap stocks (15.45%). Alternatively, the *lowest* return was produced by high-liquidity/micro-cap stocks (-.72%).

Exhibit 4.18: Heat Map of Size and Liquidity "Double Sort" Quartile Portfolios (%), Compound Annual Returns
1972–2018

Low Liquidity → High Liquidity

Micro-Cap ↓ Large-Cap				
Micro-Cap	15.45	14.97	9.08	-0.72
	15.19	13.95	11.72	5.45
	13.56	13.33	12.46	7.84
Large-Cap	11.12	11.96	11.46	8.62

Source: Compound annual returns (%) from 1972–2018. Calculated by Zebra Capital Management at www.zebracapm.com. This is an update to the research published in Ibbotson, Roger G., and Daniel Y.-J Kim, "Liquidity as an Investment Style: 2018 Update," available at www.zebracapm.com. Updated version of: Ibbotson, Roger G., Chen, Zhiwu, Kim, Daniel Y.-J., and Hu, Wendy Y. "Liquidity as an Investment Style," *Financial Analysts Journal*, May/June 2013, updated with 2013–2017 data.

In the 2018 update on liquidity, Ibbotson and Kim reach four broad conclusions: (i) liquidity should be given equal standing to size, value/growth, and momentum as an investment style, (ii) liquidity, as measured by stock turnover, is an economically significant indicator of long run returns, (iii) returns from liquidity are sufficiently different from the other styles, so that it is not merely a substitute, and finally, (iv) a stock's liquidity is relatively stable over time, with changes in liquidity associated with changes in valuation.

Ashok Abbott also investigated the relative importance of the size and liquidity risk factors.[4.30] The author used a multi-factor model including a trading cost measure and a liquidity premium factor to assess the absolute contribution for each factor individually, as well as in combination with other factors, to form an estimate of the combined contribution of the factors considered in the estimate of the cost of equity capital.[4.31]

4.30 Ashok Bhardwaj Abbott (2015). Available from the author.
4.31 A measure of an individual stock's liquidity, with higher levels signifying that the current order flow in the market can absorb larger volumes of trading without significantly affecting prices.

Abbott found significant negative relationships between the size of the companies as measured by market value of equity and his trading cost measure; stocks of larger firms can be traded at a lower cost. He found a similar relationship between liquidity and cost of trading. As stocks become more liquid, trading costs and price impact both decline, as suggested by theory.

The Risk Premium Report Study demonstrates that size and fundamental risk of small companies are correlated (discussed in chapter 10). This leads one to consider that size may, in part at best, be a coincident indicator of fundamental company risk.

That same relationship may be creating the liquidity effect. That is, the underlying risks of small companies being greater than those of larger companies may cause investors to shy away from small companies, valuing their liquidity. Thus, reduced liquidity may also be a coincident indicator of fundamental risk.

In measuring the appropriate size premium when estimating the cost of equity capital for a division or reporting unit of a large public company or a closely held business, one need not separate the portion of the size premium that may be attributable to an illiquidity factor. One is estimating the cost of capital as if the market were pricing the risks of the subject business based on the average risk of other companies of comparable size including any portion of the risks due to illiquidity.

Conclusion

The results confirm that liquidity impacts returns across styles and locations. Investing in less liquid securities generates higher returns. Liquidity seems to be an investment style that is different from size or value. This result seems to hold up in almost any equity market subset and in any location.

This section is an excerpt from a new Chartered Financial Analyst® (CFA) Institute Research Foundation monograph entitled, "Popularity: A Bridge Between Classical and Behavioral Finance" by Roger G. Ibbotson and colleagues Thomas M. Idzorek, CFA, Paul D. Kaplan, CFA, and James X. Xiong, CFA.[4.32, 4.33]

What's Next?

For many years, academics have sought to explain and understand asset prices, with a strong emphasis on market premiums and market anomalies. These premiums and anomalies can be explained by social or behavioral phenomenon in many settings. In a 2014 article, Roger Ibbotson and Tom Idzorek said, "Most of the best-known market premiums and anomalies can be explained by an intuitive and naturally occurring (social or behavioral) phenomenon observed in countless settings: popularity.[4.34]

4.32 Copyright 2018, CFA Institute Research Foundation. Reproduced from Popularity: A Bridge between Classical and Behavioral Finance with permission from CFA Institute Research Foundation. All rights reserved.

4.33 Available for download at: https://www.cfainstitute.org/en/research/foundation/2018/popularity-bridge-between-classical-and-behavioral-finance, or go to the CFA website at cfainstitute.org and search for "popularity"

4.34 Ibbotson, R.G., Idzorek, T.H. "Dimensions of Popularity," Journal of Portfolio Management, Vol. 40 No. 5, (Special 40th Anniversary Issue 2014), P. 68–74.

Popularity

The existence of various market premiums and anomalies is well established in the finance literature. To date, however, no single agreed-upon explanation for them has emerged. Investment finance is largely divided into two camps, classical and behavioral. Classical finance is based mainly on the idea that investors are risk averse, so market premiums are generally interpreted as risk premiums. In behavioral finance, premiums are considered to be the result of either cognitive errors that investors systematically make or preferences for company or security characteristics that might not be related to risks. We believe that most of the best-known market premiums and anomalies can be explained by an intuitive and naturally occurring (social or behavioral) phenomenon observed in countless settings: popularity.

What Is Popularity?

Popularity is the condition of being admired, sought after, well-known, and/or accepted. A wide range of possible categories – people, food, fashion, music, places to live, types of pet, vacation destinations, television shows, and so on – contain an implicit popularity spectrum or rank. Each of the categories has various criteria for estimating popularity.

For our purposes, the quality of the ranking criteria is not important; what is important is that any given category comprises a natural ordering in which some constituents are more popular than others. Such relative popularity evolves over time. Some aspects of popularity are systematic, or more or less permanent (for example, modern society seems to prefer thin to fat, tall to short). Other aspects of popularity may be transitory or exist only as fads (for example, necktie width, high -waisted jeans, men wearing wigs). Whether the result of systematic trends or idiosyncratic evolution, these rankings are in flux. Some popular items become relatively less popular, and some of the unpopular items become relatively more popular. While unsustainable, some popular items will temporarily become even more popular. For example, liquidity is permanently popular, but on a relative basis during times of market distress, it is especially sought after. Society places a greater relative value (monetary or otherwise) on the more popular items.

In *Popularity: A Bridge Between Classical and Behavioral Finance*, popularity refers to investor preferences – that is, how much an asset is liked or disliked. Of course, the primary preference for investors is to seek returns. Investors do not know what the returns will be, but they can distinguish one asset from another in terms of their observable characteristics, for which they may have clearly defined preferences. Thus, even with the same set of expected cash flows, investors may have more demand for one asset over another, which gives the preferred asset a higher current price and a lower expected return. An asset could be liked (or disliked) for *rational* or *irrational* reasons.[4.35] In this way, popularity spans ideas from both classical and behavioral

[4.35] Throughout *Popularity: A Bridge Between Classical and Behavioral Finance*, we describe preferences, or the reasons for preferences, as being either rational or irrational. Rational reasons for preferences are those considered in classical finance, broadly defined. The reasons include expected returns, risk liquidity, taxes, and trading costs. Generally, rational preferences are pecuniary. Irrational reasons for preferences generally are those identified in behavioral finance and result from the various biases and heuristics identified in that literature. Irrational preferences are generally nonpecuniary. Although Ibbotson, Diermeier, and Siegel (1984) acknowledged the possibility of nonpecuniary security characteristics playing a role in asset pricing (such as in the art market), their focus was on pecuniary characteristics that we consider to be subject to rational preferences. Our popularity framework extends their idea to irrational preferences.

finance, thus providing a bridge between the two camps.

In classical finance, the primary preference, beyond maximizing expected return, is to take less risk. This fact has given rise to various models that usually assume no other preferences. In the most well-known model, the capital asset pricing model (CAPM), the only "priced" characteristic is exposure to undiversifiable market risk. We consider a broader set of preferences that lead to other priced characteristics, which might include the rational preferences to reduce catastrophic losses, increase liquidity, be tax efficient, and so on. We also consider preferences that might be more in line with what the literature considers "behavioral," such as desiring to hold companies with strong brands, investments with strong past price increases, or companies that have strong ESG (environmental, social, and governance) characteristics.

The popularity framework presented in *Popularity: A Bridge Between Classical and Behavioral Finance* includes a generalization of a wide range of characteristics in classical finance and behavioral finance that influence how investors value securities. We can classify these characteristics into two broad categories with two subcategories each as follows:

Classical

- **Risks.** In classical finance, risk usually refers to fluctuations in asset values, but risk can be interpreted more broadly as any risks to which a rational investor, who assumes away any real-world frictions in the holding and trading of securities, would be averse. Thus, risks may be multidimensional, including various types of stock or bond risks, or may arise from catastrophic events.

- **Frictional.** These characteristics are often assumed away in classical finance, but a rational investor would consider them. Examples include taxes, trading costs, and asset divisibility.

Behavioral

- **Psychological.** Investors consider these characteristics because of their psychological impact. For example, buying a company with a small carbon footprint might make an investor feel good.

- **Cognitive.** Investors consider these factors or fail to accurately interpret such factors because of systematic cognitive errors. For example, investors may overvalue the importance of a company's brand when evaluating its stock because they do not realize that the value of the brand is already embedded in the market price of the stock.

Our fourfold classification of security characteristics partially overlaps with the threefold classification in Statman (2017), in which investors are described as holding securities for utilitarian, expressive, and emotional rea-sons. Utilitarian reasons correspond to risk and frictional characteristics, and expressive In and emotional reasons correspond to psychological characteristics.

In *Popularity: A Bridge Between Classical and Behavioral Finance*, we focus primarily on the stock market, although we believe the concepts can be applied to fixed-income securities, real estate, and numerous other real assets. Periodically, as necessary, we attempt to distinguish between characteristics of a company and characteristics of the security in question – both of which can have attributes that are more or less popular among investors. Assets are priced not only by their expected cash flows but also by the popularity of the other characteristics associated with the company or security. The less popular stocks have lower prices (relative to the expected discounted value of their cash flows), thus higher expected returns. Popularity can be related to risk (an unpopular characteristic), and it can also be related to other rational preferences. But popularity can also be related to behavioral concepts. For instance, investors may want to brag about their past winners (or purchase recent winners – for example, in the practice called "window dressing") or hold recognizable securities that are consistent with their social values. Any aspect that can affect the popularity of a stock will affect its demand and thus its price.[4.36]

Popularity is a bridge between classical finance and behavioral finance because both types of finance rely on preferences. Popularity is an expression of these preferences, whether they are rational, irrational, or somewhere in between.[4.37] Popularity does not make a value judgment but, instead, takes pref-erences as a given and recognizes that preferences can change over time. *Popularity: A Bridge Between Classical and Behavioral Finance* is presented in an equilibrium framework, so asset prices and expected returns reflect the aggregate impact of investor preferences.

[4.36] By demand, we mean the sum of the demand of all market participants.

[4.37] The same preference may be rational for one investor and irrational for another investor. For example, it is rational for a taxable investor to consider tax efficiency and irrational for nontaxable investor to seek out tax efficient investments.

Key Things to Remember about the Size Premium

- The size effect is based on the empirical observation that companies of smaller size are associated with greater risk and, therefore, have greater costs of capital. In other words, there is a significant (negative) relationship between the size and historical equity returns – as size *decreases*, returns tend to *increase*, and vice versa.

- Traditionally, small companies are believed to have greater required rates of return than large companies because small companies are inherently riskier. It is not clear, however, whether this is due to size itself, or to another factor closely related to size.

- The size effect is not evident just for the smallest companies; it is evident for all but the largest groups of companies, including companies with a market capitalization in excess of several billions of dollars. However, the size effect is greatest with the smallest companies.

- Small-cap companies tend to outperform large-cap companies over longer periods. The longer the period over which small-cap companies and large-cap companies are given to "race" against each other, the more likely it is that small-cap companies will outperform large-cap companies. The size effect tends to stabilize over time.

- Use sum betas for the development of size premia, and use sum beta within the CAPM (particularly if dealing with very small companies), because sum betas tend to better explain the returns of smaller companies. However, in cases in which you do use OLS betas in CAPM, you should use an OLS-beta derived size premium.

- Risk Premium Report portfolios do not include start-up and high-financial-risk companies. The returns on these companies could be expected to be high because of their risk, not because of their size.

- Despite many criticisms of the size effect, it continues to be observed in data sources. Further, observation of the size effect is consistent with a modification of the pure CAPM. Studies have shown the limitations of beta as a sole measure of risk. The size premium is an empirically derived correction to the pure CAPM.

- The 1980s were not kind to small capitalization stocks. During this period, the size effect likely was on a cyclical low, or even significantly negative.

- After the influence from the 1980s is in the rear-view mirror, small-cap companies seem to regain their footing, and the size effect seems to return to levels similar to those in the decades preceding the 1980s.

- The evidence suggests that the size effect has diminished in strength in more recent years, especially when compared to the "early years" 1926–1945, during which small-cap companies outperformed large-cap companies by a large degree.

- The size effect is still significant even after controlling for the huge small-cap outperformance of the "early years" 1926–1945.

- If the valuation analyst is estimating the cost of equity capital of a closely held subject company on an "as if publicly" basis, the valuation assumption is that the subject company would have liquidity (if it was public) to approximately the average of comparable size public companies. The size premium in the Cost of Capital Navigator are appropriate to use in developing the cost of equity capital without separating the size effect from the liquidity effect.

- The size effect is not without controversy, nor is this controversy something new. Traditionally, small companies are believed to have greater required rates of return than large companies because small companies are inherently riskier. It is not clear, however, whether this is due to size itself, or to other factors closely related to or correlated with size (e.g., liquidity).

- One can think of risk in terms of popularity. Characteristics of investments that investors *desire* are "popular", while characteristics of investments that investors do *not* desire are not popular. All other things being equal, assets with popular characteristics will be priced higher and have lower returns than assets with unpopular characteristics, which will be priced lower and have higher returns. Popularity can include all sorts of other characteristics that do not fit well into the risk and return paradigm.

- Most recently (2019), Ibbotson and colleagues Thomas M. Idzorek, CFA, Paul D. Kaplan, CFA, and James X. Xiong, CFA published a new Chartered Financial Analyst® (CFA) Institute Research Foundation monograph entitled, *Popularity: A Bridge Between Classical and Behavioral Finance* (available for download at https://www.cfainstitute.org/en/research/foundation/2018/popularity-bridge-between-classical-and-behavioral-finance).[4.38]

[4.38] Or, go to the CFA website at cfainstitute.org and search for "popularity".